# Modeling of Comfortable and Energy-Efficient Light Distributions for an Indoor Environment

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## Introduction

An indoor lighting system needs to offer adequate illuminance to satisfy multiple objectives simultaneously, such as visual comfort, visual performance and safety requirements. It has become a key challenge to increase energy efficiency without sacrificing the quality of light.

A popular and widely accepted idea is to divide the entire surface of the indoor environment into three kinds of areas, i.e. task area, surrounding area and other area. We provide enough illuminance for task area to enable users to perform their visual tasks; relatively low illuminance for other area to offer the visibility for curbs, stair edges, etc. and mid-ranged illuminance for surrounding area to avoid large variation of illuminance. In current European standard, the recommended illuminance for task area, surrounding area and other area are 500 lux, 300 lux and 60 lux, respectively [1].

However, several surveys have found that preferred light levels in working environments are often lower than recommended values [2][3]. Thus significant energy can be saved by adopting the lowest reasonable illuminance instead of the recommendations. Nevertheless, it is not easy to apply these results in an automatic control system because they don't give a formalized, computer-interpretable, mathematically unambiguous function that describes the relationship between visual comfort and illuminance. To solve this problem, this paper aims to propose a group of such functions which describes users' satisfaction for different illuminance and can be executed by an automatic controller.

This paper is organized as follows: Section 2 reviews several published experiment about user's satisfaction for different light levels. Section 3 proposes an example satisfaction function. Section 4 introduces two methods to choose the lowest illuminance requirement of a room and a method to realize a light pattern. Section 5 concludes the paper.

## Visual comfort vs. illuminance over task area

Experiments [2] and [3] indicate that every individual *i* has a preferred light level $\xi_i$. Therefore for a group of people, their preferred light levels can be regarded as a statistical random variable $\Xi$ with a certain distribution. Newham et al. asked 94 participants to set the desktop illuminance to the level that they like most and found that the preferred illuminance for most North American office workers is around 400 lux [2].

In 2001, Newsham conducted another experiment in which 47 participants were asked to set their desktop illuminance to their preferred light level [4]. Moreover, for each participant, another office worker was required to work together with the participant. After a day's work, the office worker was given an opportunity to increase or decrease the desktop illuminance ($\Delta L$) according to his/her own preference. The result of both [2] and [4] are plotted in Figure 1 which suggests that $\Xi$ is normally distributed. By maximum likelihood estimation, we found that the estimated mean value $\mu_\Xi$ is 429 lux and the estimated variance $\mu_\Xi$ is 151 lux.

Both experiments suggest that there exists a relationship between users' satisfaction and illuminance. According to [4], those office workers with small $\Delta L$ (-100 lux $\leq \Delta L \leq$ 100 lux) had a significantly higher ratings of pleasure, lighting quality and general

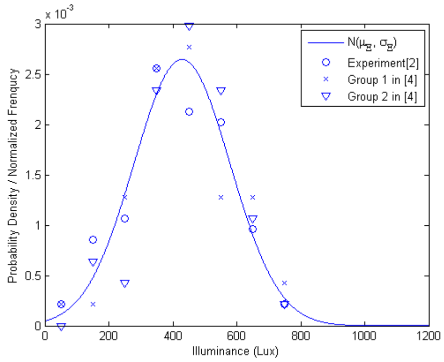


*Fig. 1: Frequency of illuminance $\xi_i$ preferred by an individual and a fitted Gaussian probability density function $f_\Xi(\xi)$.*

environmental satisfaction which means that $\Delta L$ is a good indicator for the user's opinion on the day's illuminance. Following [4], we divide the users' opinion into three categories, which are insufficient ($\Delta L$ < 100lux), satisfactory (-100lux ≤ $\Delta L$ ≤ 100lux) or excessive ($\Delta L$ < -100lux). The users' opinion of the 47 office workers are listed in Table 1 [4].

*Table 1 User's opinion of their desktop illuminance from [4]*

| Interval (lux) | Insuff. | Satis. | Excess. |
|---|---|---|---|
| 0 - 100 | 1 | 0 | 0 |
| 100 - 200 | 1 | 0 | 0 |
| 200 - 300 | 1 | 4 | 1 |
| 300 - 400 | 3 | 9 | 0 |
| 400 - 500 | 4 | 8 | 1 |
| 500 - 600 | 0 | 5 | 1 |
| 600 - 700 | 0 | 1 | 5 |
| 700 - 800 | 0 | 0 | 2 |

Thus we postulate the existence of a satisfaction function $P_i(x)$ which describes the satisfaction of individual $i$ for a certain illuminance x that is uniformly distributed. A typical satisfaction function should have following three properties:

1. $0 \le P_i(x) \le 1$, for all $x > 0$ and every user $i$;
2. Larger value of $P_i(x)$ represents higher satisfaction;
3. $P_i(x)$ has a single peak at $x = \xi_i$.

It is reasonable to assume that individual $i$ is satisfied with an interval of illuminance $\xi_{i\text{Low}} \le \xi_i \le \xi_{i\text{High}}$, where $\xi_{i\text{Low}}$ and $\xi_{i\text{High}}$ are the lowest and highest illuminance that satisfy individual $i$. Substantial amount of energy can be saved by setting the illuminance to $\xi_{i\text{Low}}$ instead of the European standard.

In order to determine $\xi_{i\text{Low}}$, we model the existence of a satisfaction threshold $\alpha_T$. An illuminance $x$ is regarded as "Satisfactory" by an individual $i$ if and only if $P_i(x) \ge \alpha_T$. Thus the probability that a randomly selected person ranks x as "Satisfactory" is'

$$
\begin{aligned}
&p(x \text{ is satisfactory}) \\
&= \mathbb{E}[p(P(x) \ge \alpha_T)],
\end{aligned}
$$

which can be calculated from $f_\Xi(\xi)$ and other parameters in $P_i(x)$. Similarly, we can derive the probability that $x$ is perceived as "Insufficient", "Satisfactory" and "Excessive". As verification, we can compare our theoretical outcome with experiment conducted by Balder in 1957 [5].

**An example $P_i(x)$**

Some people are only satisfied with a small range of illuminance while others are more tolerant, so preferably we introduce a parameter $\sigma_i$ to describe the tolerance for illuminance of each individual.

Moreover, since different individual has different maximum satisfaction, a parameter to describe the maximum satisfaction of each individual is also necessary. According to Fechner's law [6], human eye senses brightness approximately logarithmically over a moderate range. Lacking a generally accepted model for $P_i(x)$, we propose an example $P_i(x)$ as

$$P_i(x) = \alpha_i e^{-\frac{(\ln x - \ln \xi_i)^2}{2\sigma_i^2}}.$$

For a group of people, similarly as $\Xi$, both A and $\Sigma$ are treated as random variables. Since brightness perception of human being is a very complex psychological and biological process which has not been well studied yet, we make a simplification that

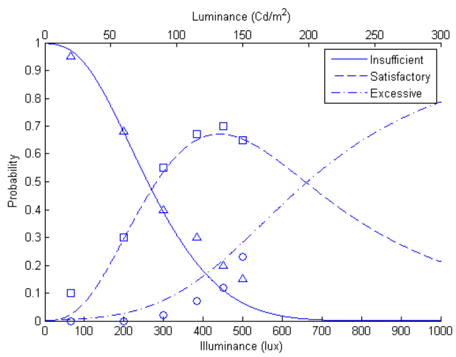


*Fig. 2: Results of our model (Illuminance) and Balder's experiment (Luminance) [5], with $\alpha_T = 0.7$, the triangles, squares and circles are Balder's result, which represent "Too dark", "Good" and "Too bright", respectively*

these three parameters are independent of each other. As discussed in Section 2, Ξ is normally distributed. However, for Σ and A, we cannot find any existing directly related experiments, so we postulate that Σ also follows a Gaussian distribution and A equals to one for all individuals. If more experiments or data are available, we can refine these assumptions.

With experimental data in Figure 1 and Table 1, we estimated the expectation and variance of Σ by maximum likelihood estimation. Then we can calculate the probability that $x$ is ranked "Insufficient", "Satisfactory" and "Excessive". The result of both our model and Balder's experiment are shown in Figure 2 as a comparison and verification.

From Figure 2, it is clearly seen that the two results have a same trend. Moreover, a good numerical match between the two results is found if the reflectance is set to 0.95. Admittedly, this is a high value for any desktop surface. However, since Balder's experiment is conducted almost 60 years ago, many other factors such as the quality of light sources, living styles, etc. have changed, which may exist an effect on the result. Expectedly, both Balder's experiments and our theoretical result indicate that it is impossible to find a light level that satisfies everyone, unless personalized settings are applied which are dependent on the users' personal preferences.

## Minimizing energy consumption

In this paper, an array of dimmable LEDs is used as light source. To minimize the energy consumption, a method based on convex optimization is applied to control the dimming matrix of the LED array. First the entire surface of the indoor environment is discretized into an $M$ by $N$ grid. Then the convex optimization problem is applied to find the optimum dimming matrix $\mathbf{W}$, as illustrated in Eq. (1).

$$\begin{aligned} \min \quad & \sum_{i=1}^{M}\sum_{j=1}^{N} I_{i,j} \\ \text{subject to} \quad & \mathbf{I} \succeq \mathbf{I_r} \\ & 0 \preceq \mathbf{W} \preceq 1 \\ & \min(\mathbf{I}_\mathrm{t}) \geq \beta_t \overline{\mathbf{I}_\mathrm{t}} \\ & \min(\mathbf{I}_\mathrm{s}) \geq \beta_s \overline{\mathbf{I}_\mathrm{s}} \end{aligned} \qquad (1)$$

In Eq. (1), $I_{i,j}$ is the illuminance at grid point $(i,j)$ which can be calculated from the dimming matrix $\mathbf{W}$ [7]; $\boldsymbol{I_r}$ is the minimum illuminance requirement; $\beta_t$, $\beta_s$ are the requirements of uniformity w.r.t the average illuminance of task area $\boldsymbol{I_t}$ and surrounding area $\boldsymbol{I_s}$, respectively. According to current European standard, $\beta_t = 0.7$ and $\beta_s = 0.5$.

$\boldsymbol{I_r}$ can be determined by either the current European standard or a method that satisfies the majority of the users. As illustrated in Figure 2, at most we can satisfy 65.29% of the users by offering 430 lux for task area. Thus we set 400 lux as the minimum requirement for task area. The corresponding minimum requirement for surrounding area can be determined according to Table 2. Other area, the walking space between office desks, should be at least one-fifth the illuminance of the floor in adjacent areas. The detailed information of both methods is listed in Table 3.

*Table 2 Relationship between illuminance of surrounding area and task area [7]*

| $I_t$ (lux) | $I_s$ (lux) |
|---|---|
| ≥750 | 500 |
| 500 | 300 |
| 300 | 200 |
| ≤200 | $I_t$ |

*Table 3 Two methods to choose $I_r$*

| | Method 1 (EU standard) | Method 2 |
|---|---|---|
| $I_t$ (lux) | 500 | 400 |
| $I_s$ (lux) | 300 | 250 |
| $I_o$ (lux) | 60 | 50 |

In order to compare the energy consumption of these two methods, we give a practical example. In this example, we consider a practical office of size 3 m by 7 m which accommodates three workers. Each of them sits behind a desk of size 0.75 m by 1.50 m, with a height of 1 m. The height of the ceiling is 2.60 m. The size of each task area is 0.5 m by 0.5 m. The lighting-on time is 11 hours per day. An array of Philips MASTER LEDspot D 7-50W 4000K PAR20 25D is used as the light source. The distance between every two LEDs in the array is 0.45 m.

The numerical result is listed in Table 4. It is clearly shown that about 20% of the energy is saved while 4% more of users are satisfied.

*Table 4 Numerical results of the two methods*

| | EU Standard | Satisfying Majority |
|---|---|---|
| Energy Consumption | 2.45 kWh/Day | 2.01 kWh/Day |
| Ave($I_t$) | 528 lux | 422 lux |
| Users satisfied | 61% | 65% |

## Conclusion & Discussion

In this paper, we propose the concept of satisfaction function, which describes the relationship between illuminance and the user's satisfaction with the lighting condition. Based on an example of our satisfaction function, we calculate the illuminance which can satisfy most of the users. Compared with the recommendations, about 20% of the energy consumption can be saved by adopting this illuminance strategy.

However, since the satisfaction function is a relatively new concept, only little experimental confirmation was available. Thus our modeling has been limited by postulated behaviors. Nonetheless a good match is found between experimental result and result derived from our model. Once more reference and experiments become available, we can refine our model to improve automatic light control system.

## References


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